

Self-consistency in the Projected Shell Model

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Abstract

The Projected Shell Model is a shell model theory built up over a deformed BCS mean field. Ground state and excited bands in even-even nuclei are obtained through diagonalization of a pairing plus quadrupole Hamiltonian in an angular momentum projected 0-, 2-, and 4-quasiparticle basis. The residual quadrupole-quadrupole interaction strength is fixed self-consistently with the deformed mean field and the pairing constants are the same used in constructing the quasiparticle basis. Taking ^{160}Dy as an example, we calculate low-lying states and compare them with experimental data. We exhibit the effect of changing the residual interaction strengths on the spectra. It is clearly seen that there are many $J^\pi = 0^+, 1^+, 4^+$ bandheads whose energies can only be reproduced using the self-consistent strengths. It is thus concluded that the Projected Shell Model is a model essentially with no free parameters.

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1 Introduction

It is known that the shell model is the most fundamental way of describing many-nucleon systems fully quantum mechanically. However, using the shell model to study deformed heavy nuclei is a desirable but very difficult task because of large dimensionality and its related problems. Therefore, for studying the heavy nuclear systems, one has had to rely mainly on algebraic or mean-field type models.

The Projected Shell Model (PSM) [1] provides one possible solution for a shell model treatment in heavy systems. In this approach, one first truncates the configuration space with guidance from the deformed mean field by selecting only the BCS vacuum plus a few quasiparticle configurations in the Nilsson orbitals around the Fermi surface, performs angular momentum projection (and particle number projection if necessary) to obtain a set of laboratory-frame basis states, and finally diagonalizes a shell-model Hamiltonian in this space. Since the deformed mean field + BCS vacuum already incorporates strong particle-hole and particle-particle correlations, this truncation should be appropriate for the low-lying states dominated by quadrupole and pairing collectivity.

Indeed, this approach has been very successful for ground band properties and near-yrast quasiparticle excitations in high-spin physics for both normally deformed [1] and superdeformed states [2] in heavy nuclei. It can describe the spectra and electromagnetic transitions quantitatively. Using microscopic degrees of freedom, it is able to explain many high spin phenomena. Very recently, attempts of using this model to study double-beta decay [3] and the physics away from the beta stability region [4] have been made.

To compare with a conventional shell model space, the PSM is a severely truncated theory with space dimension typically being 10^2 . However, the truncation in the PSM is done in a very appropriate way. We know that in treating a deformed system, a simple configuration in a conventional shell model does not correspond to any simple mode of excitation. This means that a huge configuration space would be necessary just to represent even the lowest eigenstate (the ground state) of the Hamiltonian. In the PSM, on the other hand, the ground state band is obtained by projecting on the vacuum state. Thus a single projected configuration in the PSM can be equivalent to a linear combination within a large configuration space in a conventional shell model.

Of course, one could ask a question of how effective interactions in this severely truncated theory affect the final results. In the present article, it will be shown that, while in practice very few multi-quasiparticle states are included in the diagonalization, any departure from the interactions in the Hamiltonian with self-consistent values destroys the agreement between theory and experiment. After reviewing the model and the self-consistent determination of the quadrupole-quadrupole interaction strength we calculate low-lying bands in ^{160}Dy and exhibit the effect of changing the residual interaction strengths on the spectra. It is demonstrated that most bandheads energies can only be reproduced using the self-consistent strengths. We thus conclude that the PSM has essentially no free parameters.

2 The Model

The PSM is built over a deformed mean field, which incorporates pairing effects through a Bogolyubov transformation to quasiparticle states. Both the Nilsson deformation parameter ϵ and the pairing constants G_p and G_n are taken from systematics [1]. The projected BCS vacuum gives the unperturbed ground state band in even-even nuclei, while unperturbed excited bands are obtained by projecting the multi-quasiparticle states. The multi-quasiparticle states are those with two proton and two neutron quasiparticle excitations (for even-even nuclei), with proton-neutron quasiparticle pairs (for odd-odd nuclei), or with one and three quasiparticle states (for odd nuclei). Angular momentum projection is exactly performed, providing the building blocks of the model.

A shell model Hamiltonian is diagonalized in this basis. It contains the single particle energies, monopole pairing between like particles, quadrupole-quadrupole and quadrupole-pairing interactions. The residual quadrupole-quadrupole interaction is fixed self-consistently with the deformed mean field and the pairing constants are the same used in building up the quasiparticle basis. The only remaining parameter is the one associated with the quadrupole pairing interaction, and it is set as 1/5 of the monopole-pairing constant, allowing a $\pm 10\%$ variation to adjust the position of backbending.

The Hamiltonian is explicitly written as:

$$\hat{H} = \hat{H}_0 - \frac{\chi}{2} \sum_{\mu} \hat{Q}_{\mu}^{+} \hat{Q}_{\mu} - G_M \hat{P}^{+} \hat{P} - G_Q \hat{P}_{\mu}^{+} \hat{P}_{\mu}, \quad (1)$$

where

$$\begin{aligned}\hat{H}_0 &= \sum_{\alpha} c_{\alpha}^{\dagger} \epsilon_{\alpha} c_{\alpha} \\ \epsilon_{\alpha} &= \hbar\omega \left[N - 2\kappa \hat{l} \cdot \hat{s} - \kappa\mu(\hat{l}^2 - \langle \hat{l}^2 \rangle) \right].\end{aligned}\quad (2)$$

The operators appearing in Eq. (1) are defined as

$$\begin{aligned}\hat{Q}_{\mu} &= \sum_{\alpha\beta} c_{\alpha}^{\dagger} Q_{\mu\alpha\beta} c_{\beta} \\ Q_{\mu\alpha\alpha'} &= \sqrt{\frac{4\pi}{5}} \delta_{NN'} \langle Njm | \left(\frac{r}{b}\right)^2 Y_{2\mu} | N'j'm' \rangle \\ \hat{P} &= \frac{1}{2} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\bar{\alpha}} \\ \hat{P}_{\mu}^+ &= \frac{1}{2} \sum_{\alpha\beta} c_{\alpha}^{\dagger} Q_{\mu\alpha\beta} c_{\bar{\beta}}^+.\end{aligned}\quad (3)$$

The quadrupole-quadrupole interaction strength will be discussed in next section. The monopole and quadrupole pairing interactions are given by

$$\begin{aligned}G_M &= (G_1 \mp G_2 \frac{N-Z}{A}) \frac{1}{A} \\ G_Q &= \gamma G_M.\end{aligned}\quad (4)$$

The G_M is inversely proportional to the particle number A and it contains two constants G_1 and G_2 . These constants are determined from systematics which should vary according to the size of the single particle space employed in the calculation [5].

If the multi-quasiparticle basis is expressed by $|\phi_{\kappa}\rangle$, the Hamiltonian is diagonalized in the basis spanned by $\hat{P}_{MK}^I |\phi_{\kappa}\rangle$, where \hat{P}_{MK}^I is the angular momentum projection operator. This lead to the eigenvalue equation:

$$\sum_{\kappa'K'} (H_{\kappa K \kappa' K'}^I - E N_{\kappa K \kappa' K'}^I) F_{\kappa' K'}^I = 0 \quad (5)$$

with the normalization condition:

$$\sum_{\kappa K \kappa' K'} F_{\kappa K}^I N_{\kappa K \kappa' K'}^I F_{\kappa' K'}^I = 1, \quad (6)$$

where

$$\begin{aligned}H_{\kappa K \kappa' K'}^I &= \langle \phi_{\kappa} | \hat{H} \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle \\ N_{\kappa K \kappa' K'}^I &= \langle \phi_{\kappa} | \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle.\end{aligned}\quad (7)$$

The normalized eigenstate is given by:

$$|\Psi_{IM}\rangle = \sum_{\kappa K} F_{\kappa K}^I \hat{P}_{MK}^I |\phi_{\kappa}\rangle. \quad (8)$$

3 Mean field deformation and the quadrupole-quadrupole force

The logical structure of the theory is the following:

- a) The Nilsson Hamiltonian is diagonalized for a given deformation.
- b) The Bogolyubov transformation is performed in order to take into account the static monopole pairing force. Three major oscillator shell are included for protons and other three for neutrons. It defines the Nilsson + BCS quasiparticle basis.
- c) The Hamiltonian is then diagonalized within the shell model space spanned by a selected set of projected multi qp-states. For even-even nuclei we chose:

$$|0 >, a_{\nu_1}^+ a_{\nu_2}^+ |0 >, a_{\pi_1}^+ a_{\pi_2}^+ |0 >, a_{\nu_1}^+ a_{\nu_2}^+ a_{\pi_1}^+ a_{\pi_2}^+ |0 > . \quad (9)$$

The effect of rotation is described by the projection operator and the whole dependence of the wave functions on spin is contained in the eigenvectors since the quasiparticle basis is spin independent. It is this feature which makes not only the treatment simple and stable but also the interpretation of the result easy and intuitive.

The quadrupole-quadrupole interaction strength is obtained identifying the Hartree potential resulting from Hamiltonian (1) with the Nilsson stretched potential

$$\begin{aligned} H_{Nilsson}^\alpha &= H_0^\alpha - \frac{2}{3}\epsilon_\alpha \hbar \omega_\alpha \hat{Q}_0, & \alpha &= p, n, \\ \omega_\alpha &= \omega_0 \left\{ 1 \pm \frac{N-Z}{A} \right\}^{\frac{1}{2}}, & \hbar \omega_0 &= 41.47 A^{-\frac{1}{3}} \text{ MeV}, \end{aligned} \quad (10)$$

with the $+$ ($-$) sign holding for $\alpha = \text{neutron}$ (proton).

The HFB single-particle Hamiltonian which results from Eq. (1) is:

$$\hat{H} = \hat{H}_0 - \chi < \hat{Q}_0 > \hat{Q}_0 - G < \hat{P} > (\hat{P} + \hat{P}^+). \quad (11)$$

Equating the first two terms in (11) with the Nilsson Hamiltonian (10) we obtain the self-consistent condition

$$\begin{aligned} \chi_{nn} < \hat{Q}_0 >_n + \chi_{np} < \hat{Q}_0 >_p &= \frac{2}{3} \hbar \omega_n \epsilon, \\ \chi_{pp} < \hat{Q}_0 >_p + \chi_{pn} < \hat{Q}_0 >_n &= \frac{2}{3} \hbar \omega_p \epsilon \end{aligned} \quad (12)$$

where the same deformation ϵ was used for protons and neutrons.

Assuming a pure isoscalar coupling [1], i.e.

$$\chi_{np} = \chi_{pn} = \chi_{pp} \frac{\omega_n}{\omega_p} = \chi_{nn} \frac{\omega_p}{\omega_n} \quad (13)$$

the above equations determine the Q-Q coupling constants χ as a function of ϵ :

$$\chi_{\alpha\alpha'} = \frac{\frac{2}{3}\epsilon\hbar\omega_\alpha\hbar\omega_{\alpha'}}{\hbar\omega_n < \hat{Q}_0 >_n + \hbar\omega_p < \hat{Q}_0 >_p} \quad (14)$$

The expectation values $< \hat{Q}_0 >_n$ and $< \hat{Q}_0 >_p$ are evaluated adding the quadrupole moments of all active deformed Nilsson orbitals weighted by their occupation numbers. They depend, of course, on the assumed mean field deformation ϵ and vanish for spherical nuclei, leaving χ undetermined in that case.

4 Variation of the parameters

While in the previous sections it was argued that all the parameters in the Hamiltonian (1) are fixed, either by self-consistency (quadrupole-quadrupole) or by systematics (mean field deformation, pairing), it is well known that the truncation of the Hilbert space affect those parameters. As an example, when in RPA calculations the “physical” quadrupole operator is replaced by its “algebraic” counterpart, which do not mix different major shells, the self-consistent χ is usually enlarged by a factor of two to compensate for this truncation in order to get the right spectra [6]. In this section we will study the influence on the spectra when the residual interaction strengths depart from their self-consistent values.

With the value of the quadrupole-quadrupole strength χ self-consistently determined from Eq. (14), we introduce two adimensional parameters x and g in the Hamiltonian, which now looks like

$$\hat{H} = \hat{H}_0 - x \frac{\chi}{2} \sum_{\mu} \hat{Q}_{\mu}^{+} \hat{Q}_{\mu} - g G_M \hat{P}^{+} \hat{P} - G_Q \hat{P}_{\mu}^{+} \hat{P}_{\mu}. \quad (15)$$

When $x = g = 1$, we recover the original Hamiltonian (1).

In what follows we will present calculations for ^{160}Dy . The BCS equations were solved for Nilsson single particle energies including the $N = 3, 4, 5$ major shells for protons and the $N = 4, 5, 6$ for neutrons. The reported deformation $\epsilon = 0.25$ [7] is used, but some results calculated with $\epsilon = 0.29$ are also presented. Proton and neutrons quasiparticle pairs with energies lower than 1.5 MeV, as well as states with two proton and two neutron quasiparticles with energies lower than 3.0 MeV were included in conjunction with the BCS vacuum to build up the truncated Hilbert space.

In Fig. 1, the band-head energies are plotted against x , when no residual pairing and quadrupole-pairing interactions are present ($g = 0$ in (15), $\gamma = 0$ in (4)). The notation used to classify the states is J_i , where J is the bandhead angular momentum and i enumerates the states with the same J in order of increasing energy. On the right corner of Fig. 1, the experimental energies are given. It is clearly seen that there is only a narrow region around $x \approx 1.0$ where all the states are compressed, in good agreement with the experimental data. Beyond this region the energies of two excited 0^+ and two 4^+ states grow more than four times, providing the image of a valley whose bottom reproduce the experimental energies. The first excited 0^+ state (0_2) is nearly insensitive to changes in x . The first 1^+ state exhibit a change in slope when $x \approx 1.3$. In ^{160}Gd the bandheads with $J = 1$ and $J = 4$ are nearly degenerated, and there is some consensus that they are built mainly from the same qp pair, which has $k_1 + k_2 = 4$, $|k_1 - k_2| = 1$. Only in a small region around $x \approx 1.0$ these two states (1_1 and 4_2) remain close to each other. In this region their dominant components are the two quasi-neutron states $[5 f7/2 -3/2]n$ and $[5 h9/2 5/2]n$.

In Table 1 we list the main components of each bandhead state for four values of x . It is remarkable that only for $x = 1$ all of them can be described as two quasiparticle bands, while in all the other cases some states have important four quasiparticle components. Notice also that for small x values there is a crossing between 4_1 and 4_2 and their interchange their components.

The first 2^+ state is usually assigned as the bandhead of the gamma band, which is interpreted as a rotor band built over a one-phonon quadrupole vibration. In Fig. 1, it can be seen that the first 2^+ state has a complicated behaviour and its energy at $x = 1$ is nearly twice the experimental value $E_\gamma(2^+)_{exp} = 0.974$ MeV. This results is not privative of ^{160}Dy . In many other deformed nuclei the PSM predicts a first 2^+ band too high in energy [1]. For $x > 1.4$ the energy of this state is decreasing, and around $x \approx 2.4$ it

would become the ground state. A microscopic analysis of the gamma band is presented in the following section.

In order to check the validity of the above mentioned results we have performed a similar calculation with a different deformation. We chose $\epsilon = 0.29$ rather than the reported $\epsilon = 0.25$ which provides a better fit of the rotational ground state band. We will discuss the rationale of selecting a larger deformation for this particular nucleus in section 6. By the moment the relevant point is to confirm that only around the self-consistent values $x = 1$ the bandheads energies reproduce their experimental values. They are plotted in Fig. 2 against x , again for $g = 0, \gamma = 0$. We found that curves are similar to those shown in Fig. 1, exhibiting a deep valley. The gamma band is still too high, but now the first excited 4^+ state is going down in energy faster than the 2^+ for $x > 1.2$. The origin of this difference can be traced back to the quasiparticle content of both bands. A change in deformation implies a different active valence particle region, with other two quasiparticle pairs commanding the behaviour of the low energy apart of the spectra.

In Fig. 3 we present a similar analysis including both monopole and quadrupole pairing ($g = 1, \gamma = 0.18$). It shows that the inclusion of the remaining interactions do not introduce significant changes. It means that the gross structure of the bandheads is due to the mean field and quadrupole-quadrupole interaction and that the residual pairing and quadrupole-pairing interactions just induce minor rearrangements in the bandhead positions. Pairing strengths significantly influence the level spacing within a band.

If isovector couplings are allowed, violating relations (13), a similar trend is found. It is shown in Fig. 4, where we kept χ_{pp} and χ_{nn} at their self-consistent values but renormalized $\chi_{pn} = x_3 \chi_{pn}^0$. The energy variations are smaller than in the previous figures because we are only changing one component of the quadrupole-quadrupole force.

Fig. 5 shows the bandhead energies as function of the pairing strength parameter g for $x = 1, \gamma = 0.18$. All curves are smooth and grow monotonically with g . It provides additional evidence about the dominant role of the residual quadrupole-quadrupole force in shaping the spectra, while the residual pairing interaction has a very modest effect in the low energy spectra.

5 The gamma band

The major components $F_{\kappa K}^{I=2}$ of the gamma bandhead are plotted in Figs. 6 and 7 against their two-quasiparticle energies for $x = 0.7$ and 1.4 respectively. Near each line the pair angular momentum projection in the intrinsic system $K = k_1 \pm k_2$ is given, as well as the two quasiparticle pairs they are built of. Notice that the components $F_{\kappa K}^{I=2}$ of the wave functions (8) are not orthonormal in the usual sense, but satisfy the orthonormality relationships (6), where the overlap matrix (7) can have very small components. For this reason the wave function components can be larger than one, and the relevant information in Figs. 6 and 7 lies in the relative magnitudes and phases.

For $x = 0.7$ a single quasiparticle pair ($[6 \text{ i}13/2 \text{ } 1/2]n$ - $[6 \text{ i}13/2 \text{ } -3/2]n$) with $K = 2$ with qp-energy slightly smaller than 1 MeV dominates the 2^+ state which is the head of the gamma band, with small mixing with two $K = 0$, two $K = 1$ and another $K = 2$ quasiparticle pairs. In contrast, for $x = 1.4$ we have at least nine $K = 2$ qp pairs which contribute, mainly in a coherent way, to the gamma bandhead wave function.

Qualitatively we would expect the gamma band to have a collective character, as that found for $x = 1.4$. On the other side, the energy of the gamma bandhead is always too high, as pointed out in the previous section. Enlarging the Hilbert space does not change this result. It seems that the wave function has enough collectivity but the Hamiltonian is not rich enough to fit its energy correctly.

In Fig. 8 the variation of the energy of the gamma bandhead is presented as function of the mean field deformation for $0.16 \leq \epsilon \leq 0.32$. All the other Hamiltonian parameters were fixed self-consistently. It can be seen that the energy of this $J = 2$ state decreases in a smooth monotonic way, but the change is too small to adjust the experimental energy.

The above discussion about the gamma bandhead leads us to conclude that there is a clear need to include additional degrees of freedom in Hamiltonian (1) in order to allow the PSM to describe such fine details of the spectra. Changes in the strength of the quadrupole-quadrupole or pairing forces would have only strongly negative effects on the general description of the spectra. Through many studies in the past, the presence of these forces in a Hamiltonian has been found to be very important, if not sufficient, to explain a large body of data. They simulate the essence of the most important correlations in nuclei, so that even the realistic force has to contain at least

these components implicitly in order for it to work successfully [8]. Of course, the simple pairing plus quadrupole forces cannot describe everything that is taken place in many-nucleon systems and thus have to be supplemented with other types of interaction whenever necessary. Spin dependent two body interactions, which have important contributions in other microscopic theories [10], could be possible candidates to enrich the PSM Hamiltonian.

6 The ^{160}Dy yrast band

The PSM does an impressive job in describing the spectra and electromagnetic transitions in heavy deformed [1] and superdeformed nuclei [2]. However, for the Dy isotopes some discrepancies has been found in previous works[9].

We present in Fig. 9 the alignment diagram of the yrast band in ^{160}Dy . It plots energy differences $\omega = (E(J) - E(J-2))/2$ against the angular momentum J , for two different deformations $\epsilon = 0.25$ and 0.29 . The experimental data are shown as a thick line. It is apparent that the larger deformation is able to reproduce the data.

The backbending plot showed in Fig. 10 reinforces this conclusion. The graph presents twice the moment of inertia Θ [1] as function of ω^2 for the same two deformations. The backbending is exaggerated for $\epsilon = 0.25$ and well reproduced by the smoother curve with $\epsilon = 0.29$. Therefore, an 15 % increase in the deformation allows the PSM to match the experimental data. It deserves a closer and more systematic analysis over many isotopes, including B(E2) transitions, which will be presented elsewhere [11].

7 Summary

Properties of the spectra of ^{160}Dy in the Projected Shell Model were studied allowing an artificial variation of the parameter strengths from their self-consistent values (in the case of the quadrupole-quadrupole interaction) or their values taken from systematics (Nilsson mean field deformation and pairing).

The dominant role of the quadrupole-quadrupole force in deformed nuclei was once again confirmed. It was clearly exhibited that any departure of its

strength from the value obtained self-consistently from the mean field deformation completely destroy the agreement with experimental data, because many excited bands increase drastically their energies. It was concluded that, even when a strongly truncated quasiparticle space is in use, the quadrupole-quadrupole force in the PSM is completely determined by self-consistency. It is worth mentioning that the same self-consistent approach has been also fruitful to remove spurious states in the RPA treatment of deformed nuclei [12].

The residual pairing interaction between quasiparticles has only a smooth effect over the spectra. It provides a better description of excited bands but is just a second order correction.

The mean field deformation ϵ is in principle fixed by systematics [7]. However, we found that, for ^{160}Dy , using a 15 % larger deformation provides a notably improved description of backbending. A detailed study of the spectra and electromagnetic transitions in Dy isotopes using a larger deformation will be presented in another article.

We also found the limitations of the PSM in the description of the gamma band. It was demonstrated that Hamiltonian (1) would need to be improved to be able to correctly describe the energetics of these states. Inclusion of a spin-dependent interaction could work in this direction.

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Figure Captions

Figure 1: Bandheads energies vs. x for $\epsilon = 0.25, g = 0, \gamma = 0$. The notation J_i is used to indicate the angular momentum J and the order i , in increasing energy, of each state. On the right hand side the experimental energies are drawn.

Figure 2: Bandheads energies vs. x for $\epsilon = 0.29, g = 0, \gamma = 0$.

Figure 3: Bandheads energies vs. x for $\epsilon = 0.29, g = 1.0, \gamma = 0.18$.

Figure 4: Bandheads energies vs. x_3 for $\epsilon = 0.25, g = 1.0, \gamma = 0.18$.

Figure 5: Bandheads energies vs. g for $\epsilon = 0.29, x = 1.0, \gamma = 0.18$.

Figure 6: Gamma bandhead main components for $x = 0.7$ as a function of the 2qp-energy. The angular momentum projection $K = k_1 \pm k_2$ is given near each line, together with the main 2 qp components. 4 qp means that that a four quasiparticle pair dominates the state.

Figure 7: Gamma bandhead main components for $x = 1.4$ as a function of the 2qp-energy.

Figure 8: Gamma bandhead energy vs. ϵ .

Figure 9: Angular frequencies versus the angular momentum in ^{160}Dy .

Figure 10: The moment of inertia as function of the squared angular frequency for ^{160}Dy .

Table 1: Dominant spherical quasiparticle component of each bandhead state for 4 values of x . The x values, J_i bandhead classification, energies of the 2-qp pairs, K angular momentum projection and the spherical labels of the qp pairs are listed.

x	Bandhead	Pair Energy	Projection	Spherical labels [N L J M] T_3
0.0	0_2	3.6525	0	4qp
	0_3	2.2842	0	[5 h11/2 5/2]p [5 h11/2 5/2]p
	0_4	1.4283	0	[5 f7/2 -3/2]n [5 f7/2 -3/2]n
	1_1	3.8841	-1	4qp
	2_2	2.4481	2	[6 i13/2 1/2]n [6 i13/2 -3/2]n
	4_2	1.7536	-4	[6 i 13/2 3/2]n [6 i13/2 5/2]n
	4_3	1.4204	-4	[5 f7/2 -3/2]n [5 h9/2 5/2]n
0.7	0_2	3.6525	0	4qp
	0_3	1.8315	0	[6 i13/2 -3/2]n [6 i13/2 -3/2]n
	0_4	1.4283	0	[5 f7/2 -3/2]n [5 f7/2 -3/2]n
	1_1	3.8841	-1	4qp
	2_2	2.4481	2	[6 i13/2 1/2]n [6 i13/2 -3/2]n
	4_2	1.4204	-4	[5 f7/2 -3/2]n [5 h9/2 5/2]n
	4_3	1.7536	-4	[6 i13/2 -3/2]n [6 i13/2 5/2]n
1.0	0_2	1.8211	0	[5 h11/2 -7/2]p [5 h11/2 -7/2]p
	0_3	1.4126	0	[5 h9/2 5/2]n [5 h9/2 5/2]n
	0_4	1.6758	0	[6 i13/2 5/2]n [6 i13/2 5/2]n
	1_1	1.4204	1	[5 f7/2 -3/2]n [5 h9/2 5/2]n
	2_2	2.0985	-2	[4 d5/2 -3/2]p [4 d3/2 1/2]p
	4_2	1.4204	-4	[5 f7/2 -3/2]n [5 h9/2 5/2]n
	4_3	1.7536	-4	[6 i13/2 -3/2]n [6 i13/2 5/2]n
1.4	0_2	3.4968	0	4qp
	0_3	1.8315	0	[6 i13/2 -3/2]n [6 i13/2 -3/2]n
	0_4	3.6525	0	4qp
	1_1	3.7284	-1	4qp
	2_2	4.0111	2	[6 i13/2 5/2]n [6 d5/2 1/2]n
	4_2	1.4204	-4	[5 f7/2 -3/2]n [5 h9/2 5/2]n
	4_3	3.5747	-4	4qp

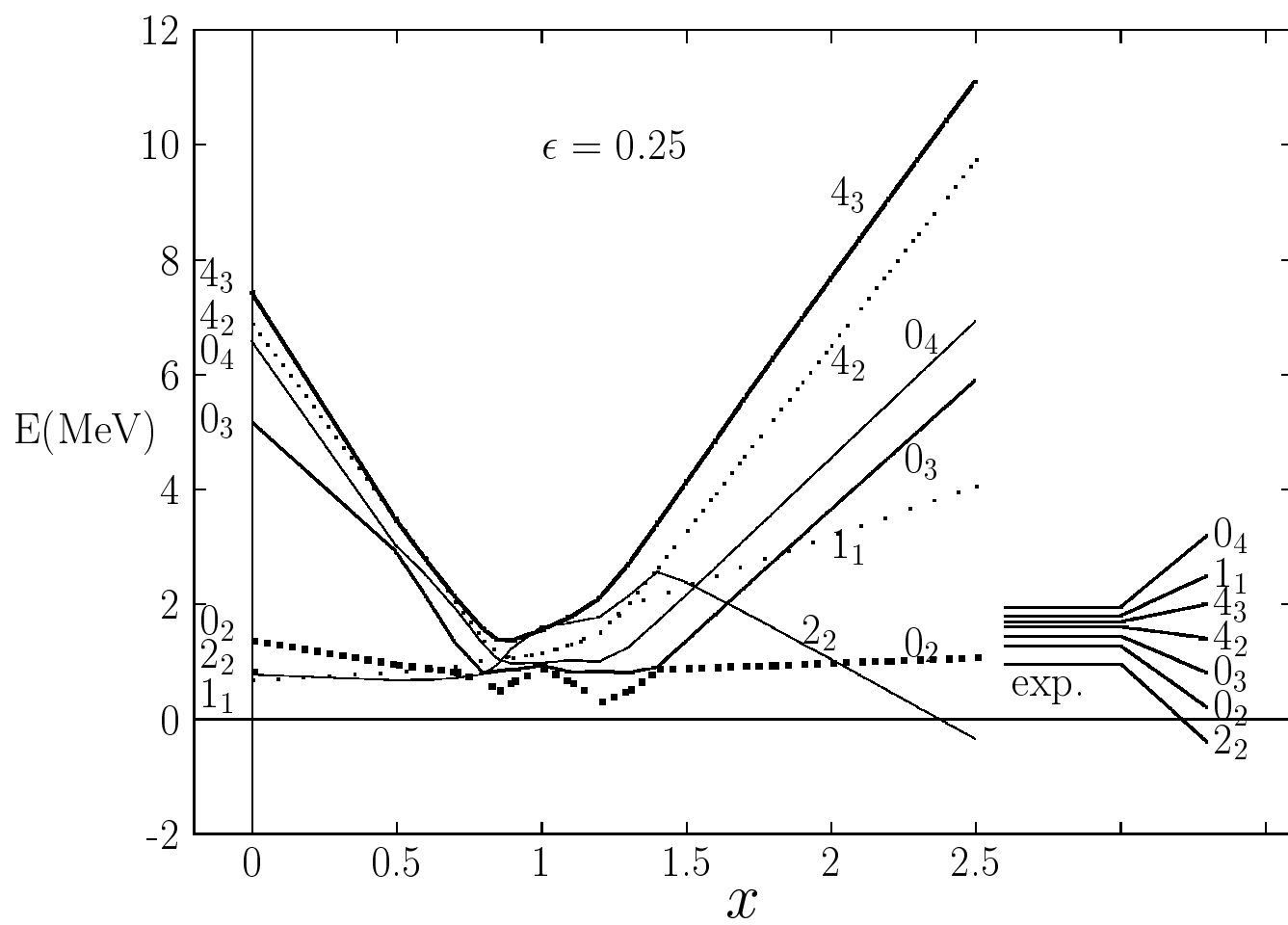


Figure 1:

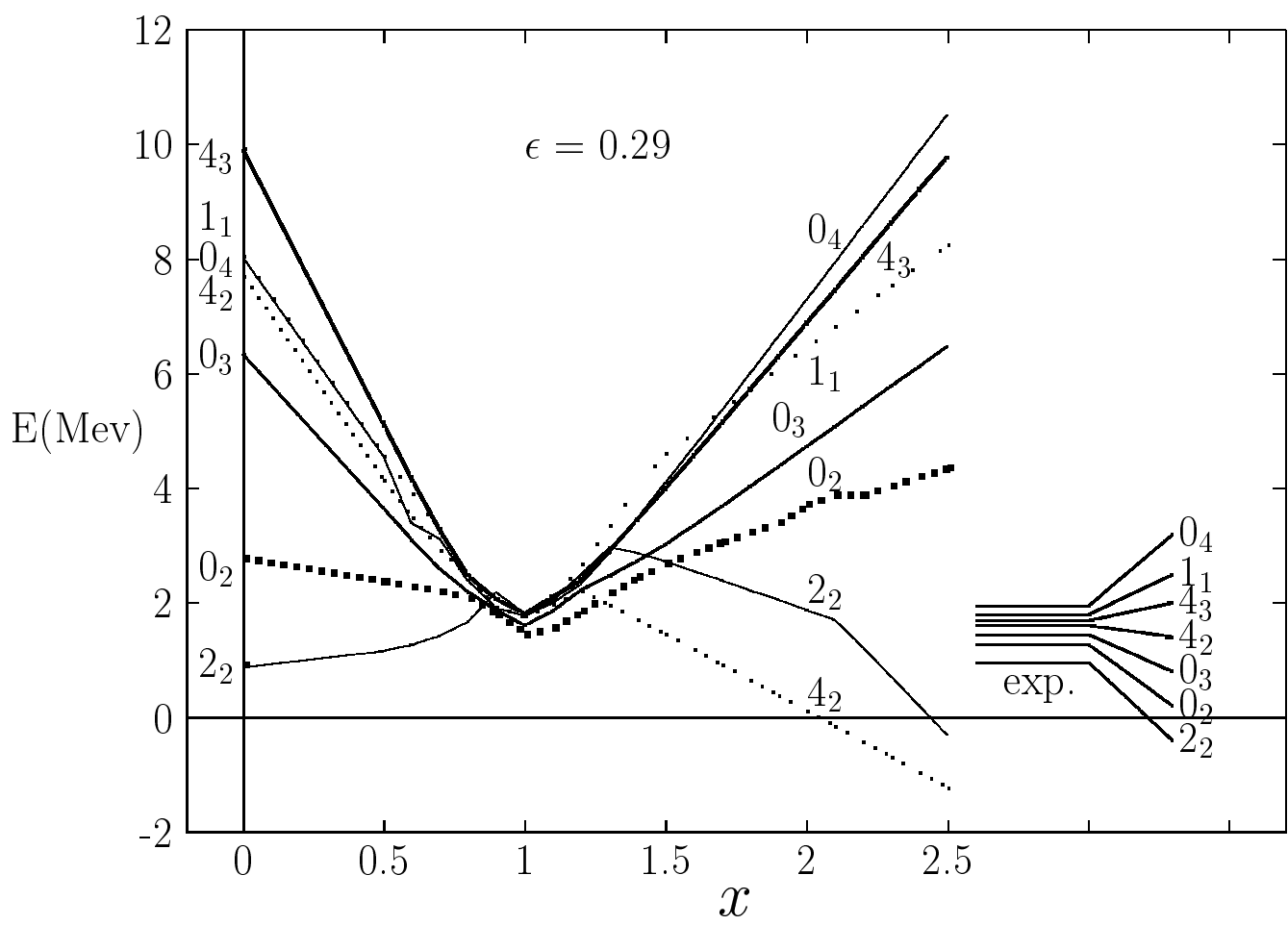


Figure 2:

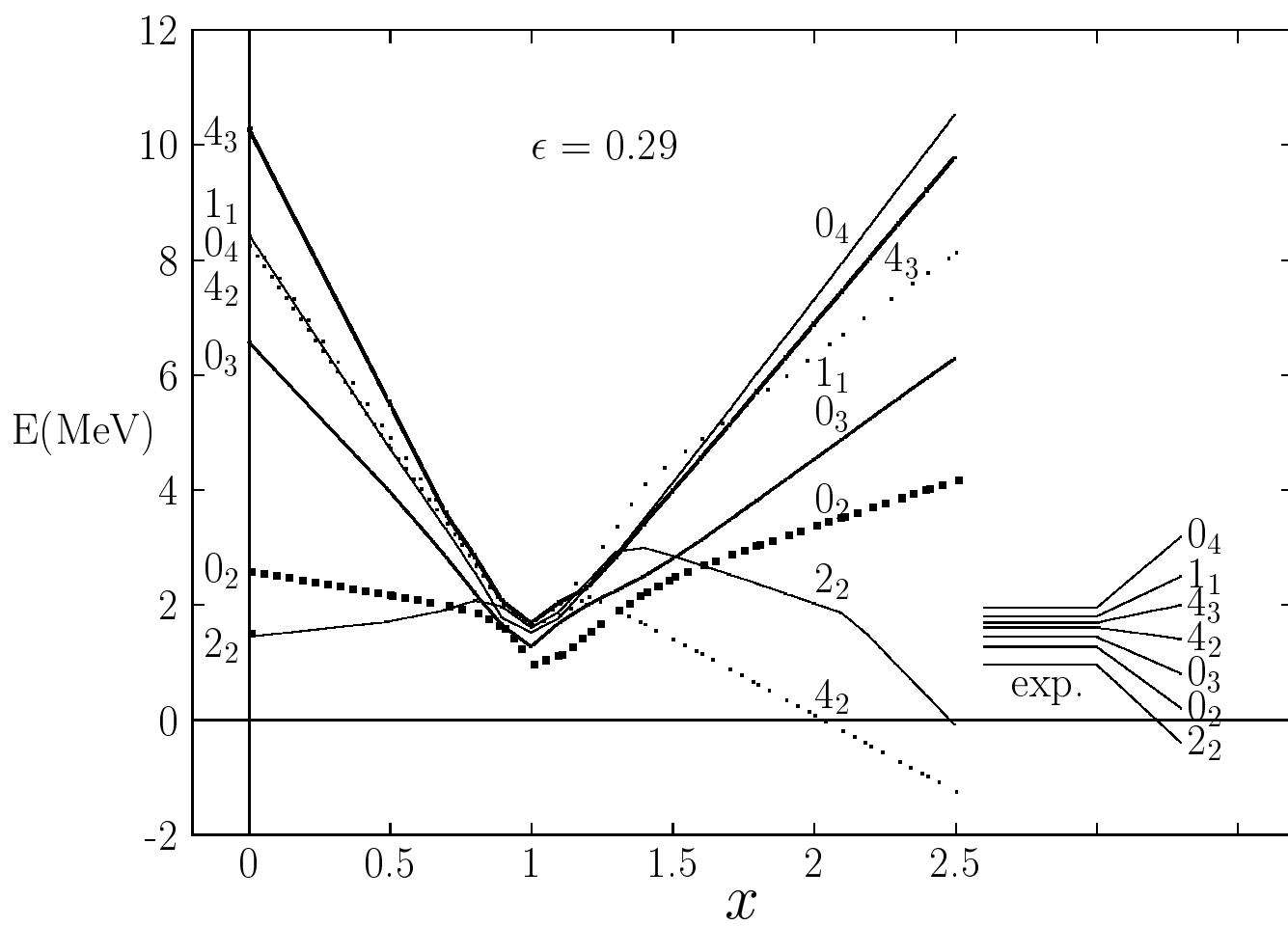


Figure 3:

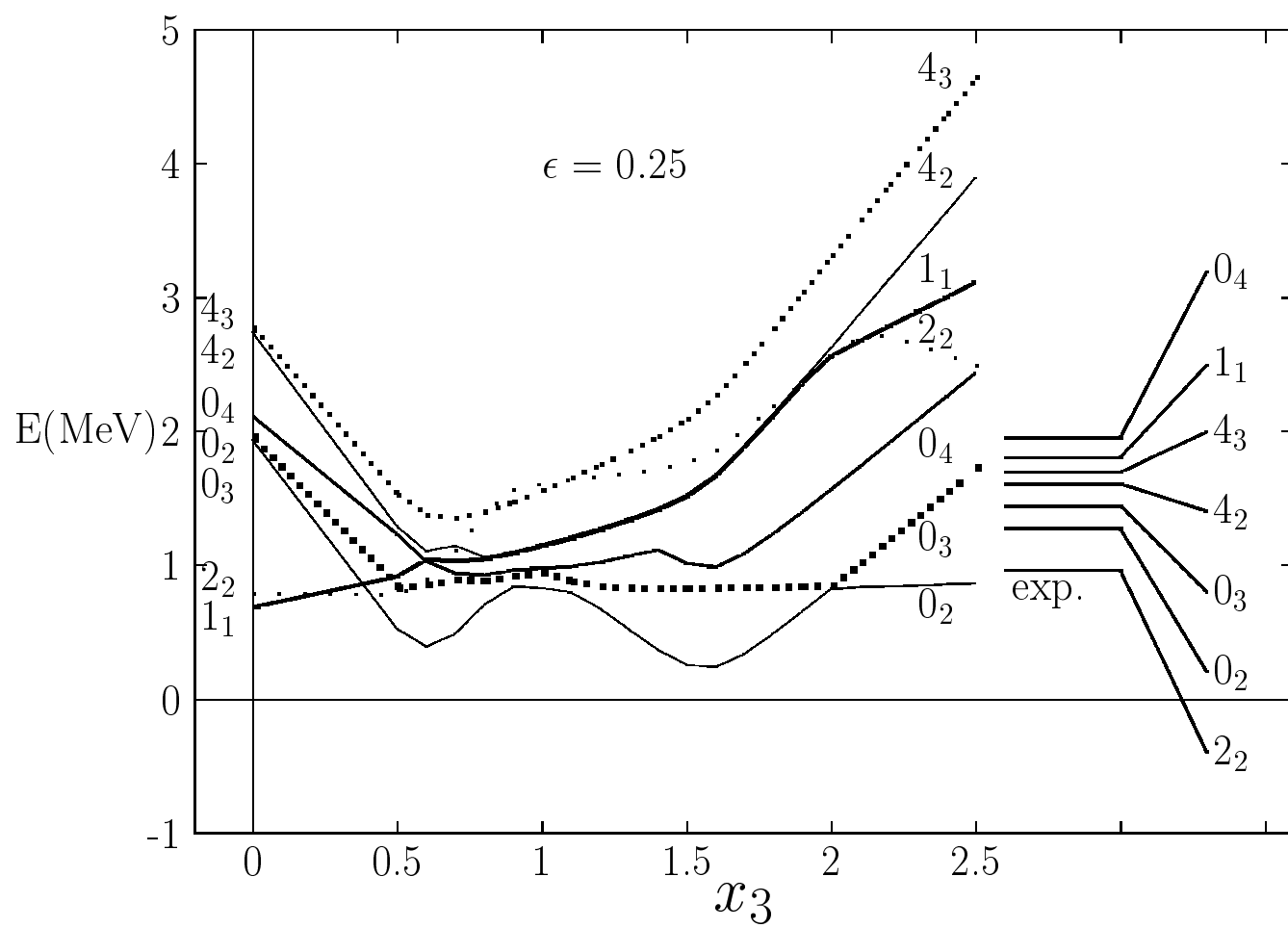


Figure 4:

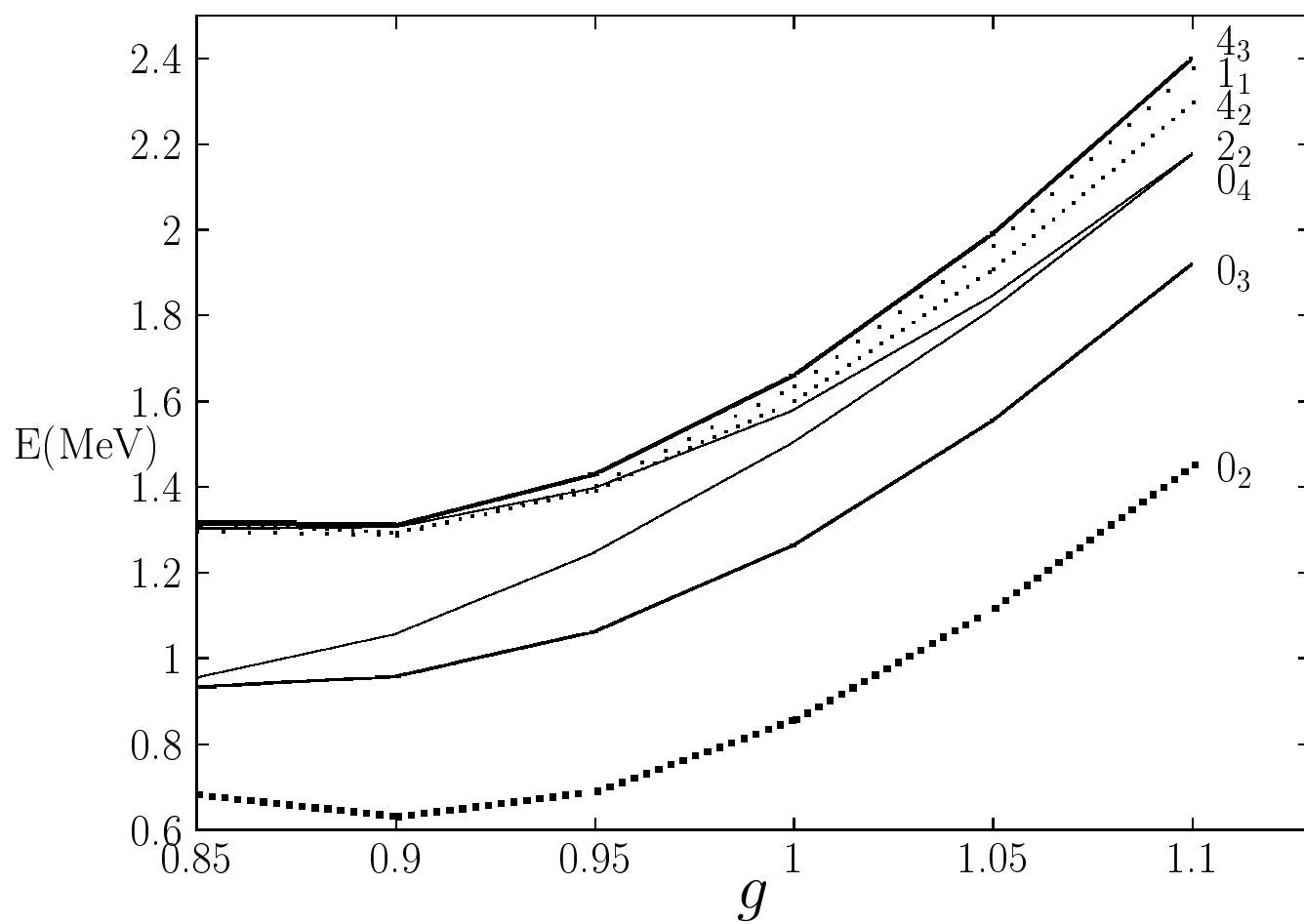


Figure 5:

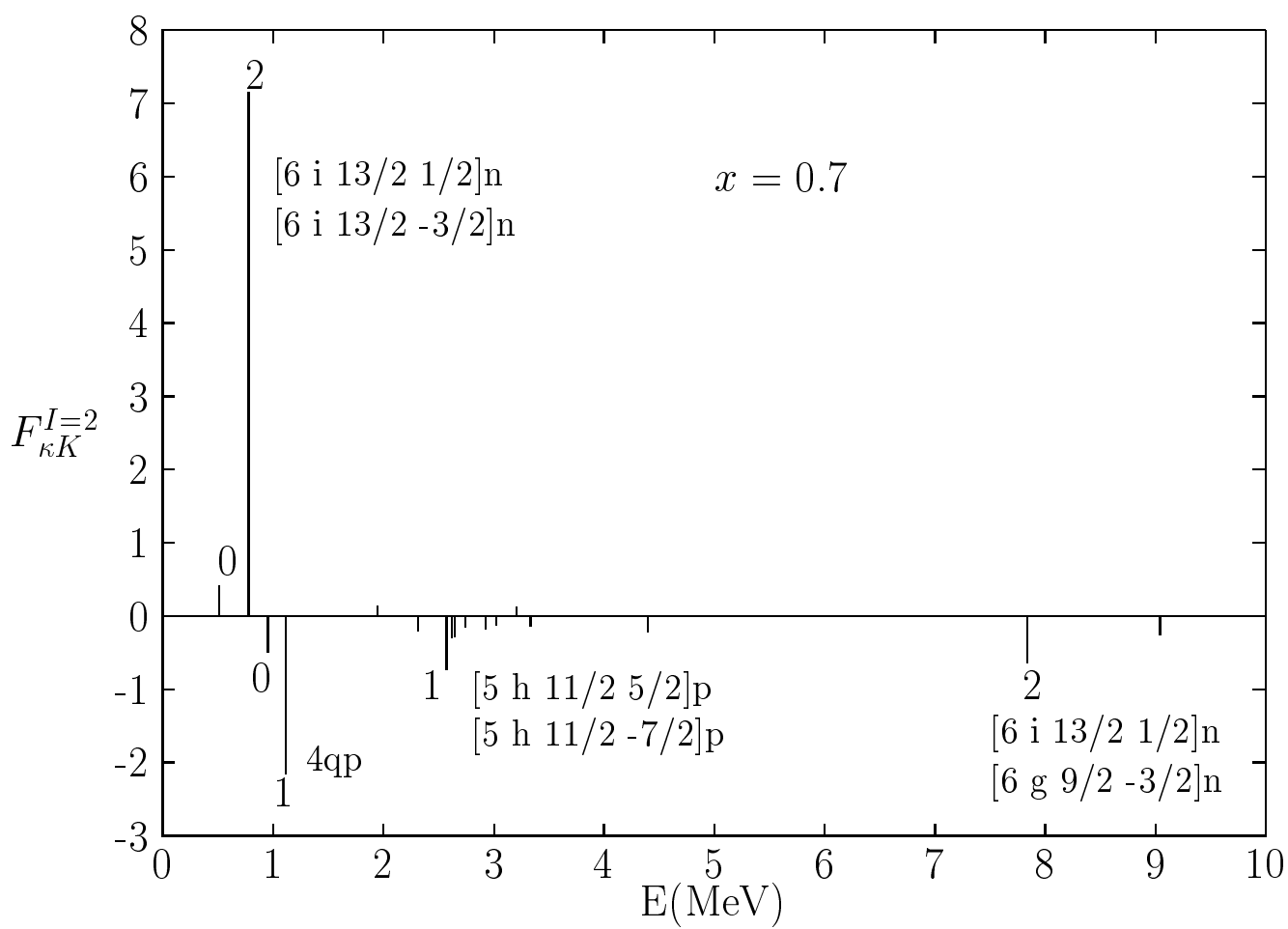


Figure 6:

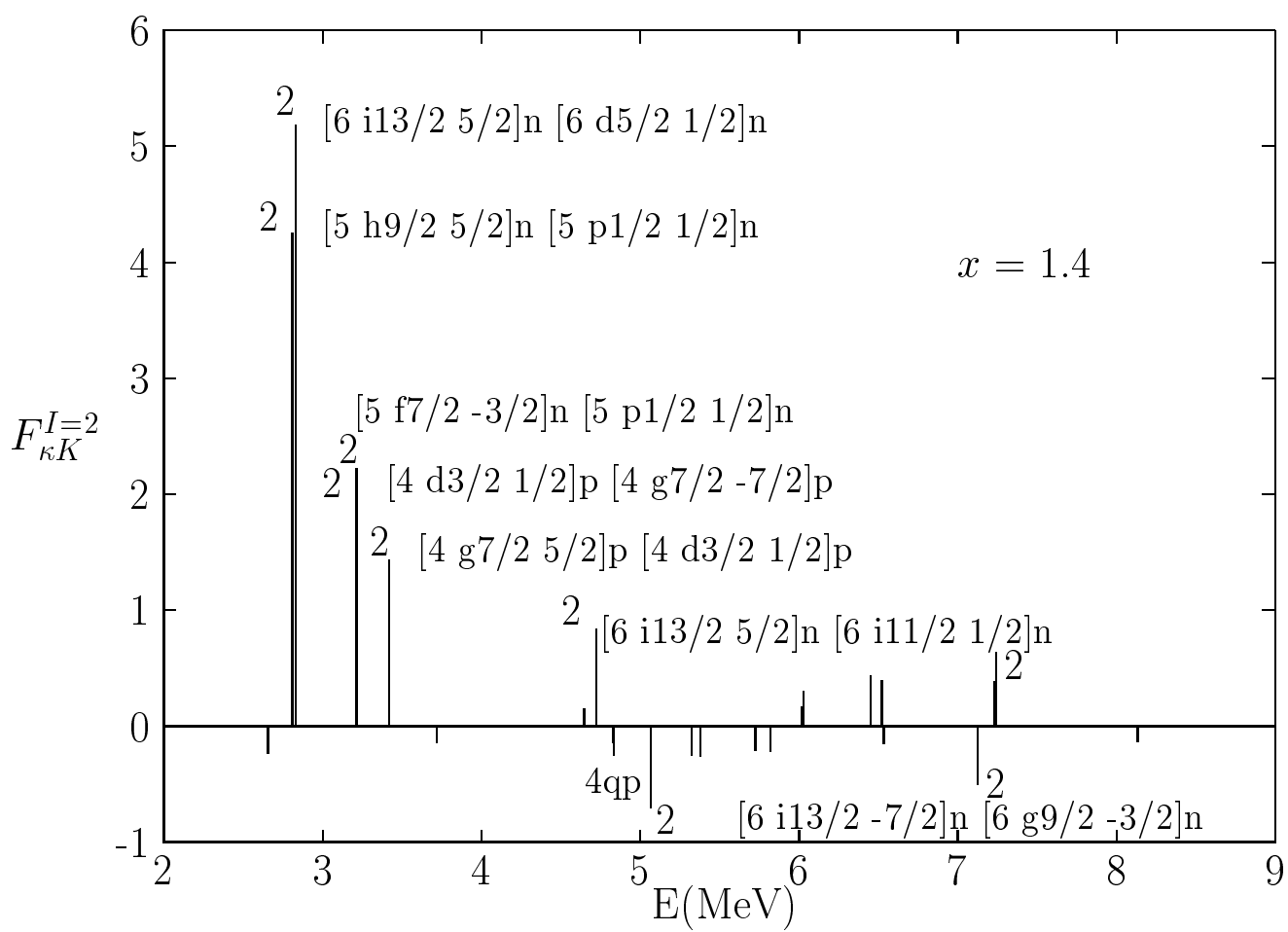


Figure 7:

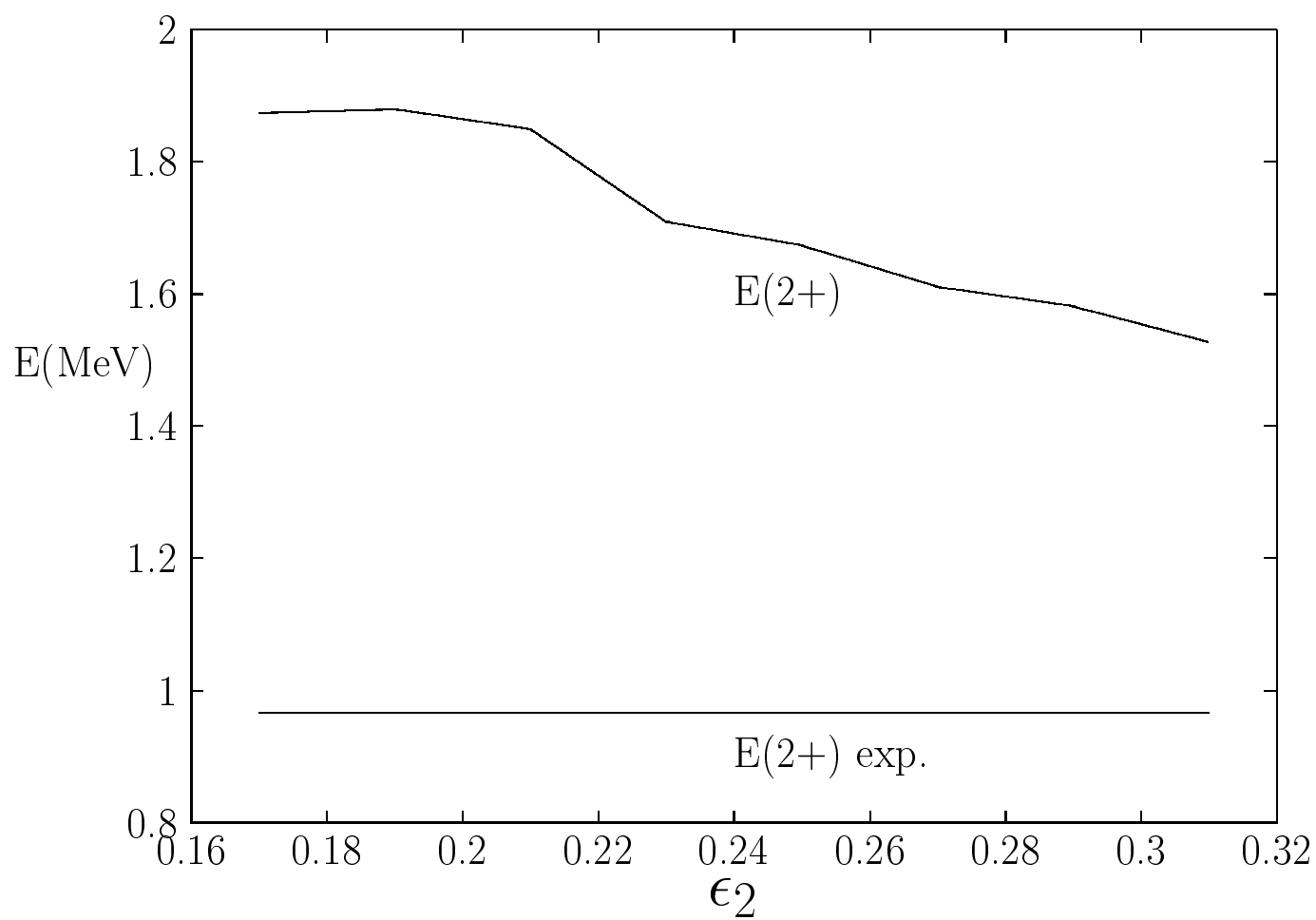


Figure 8:

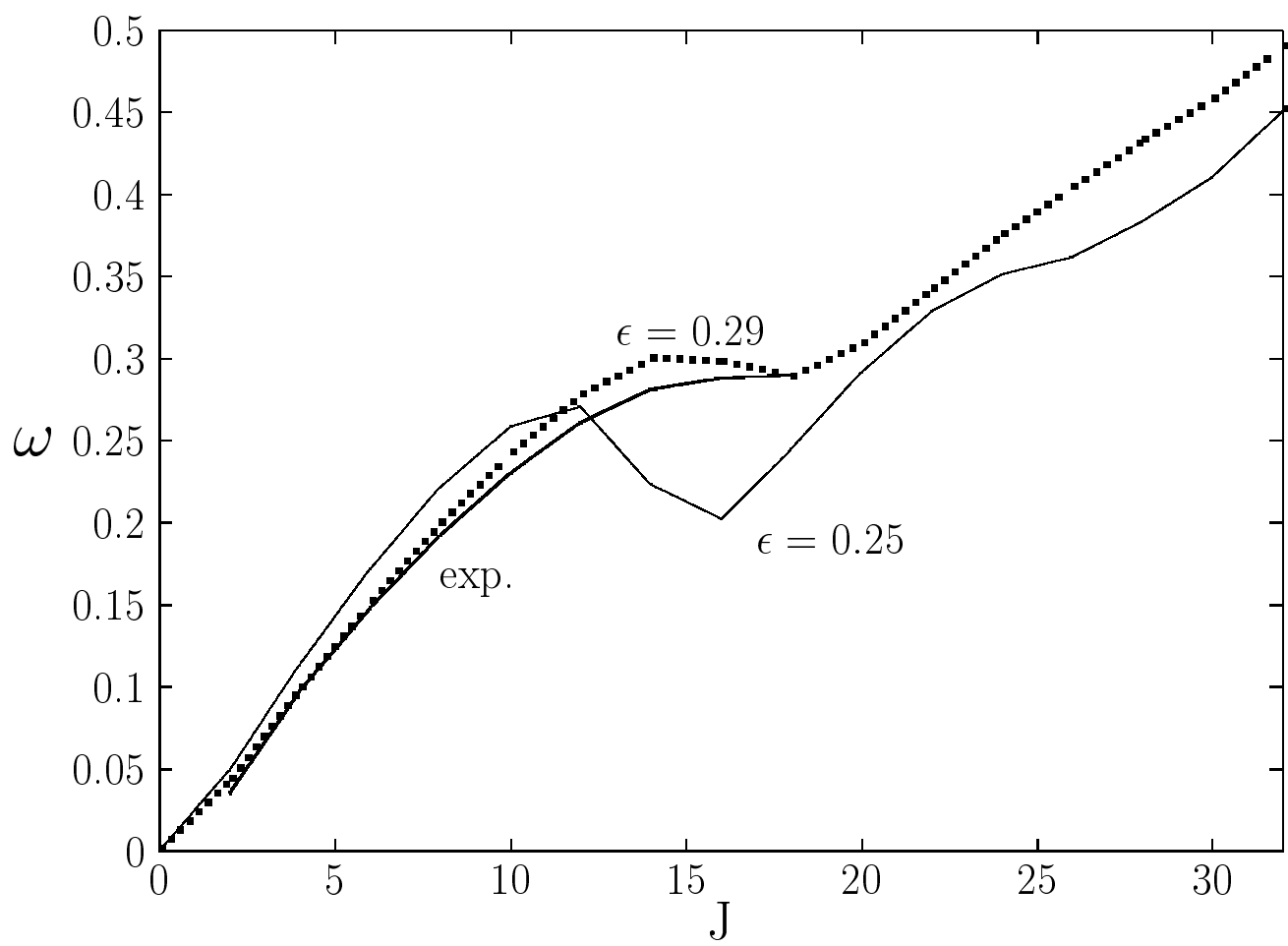


Figure 9:

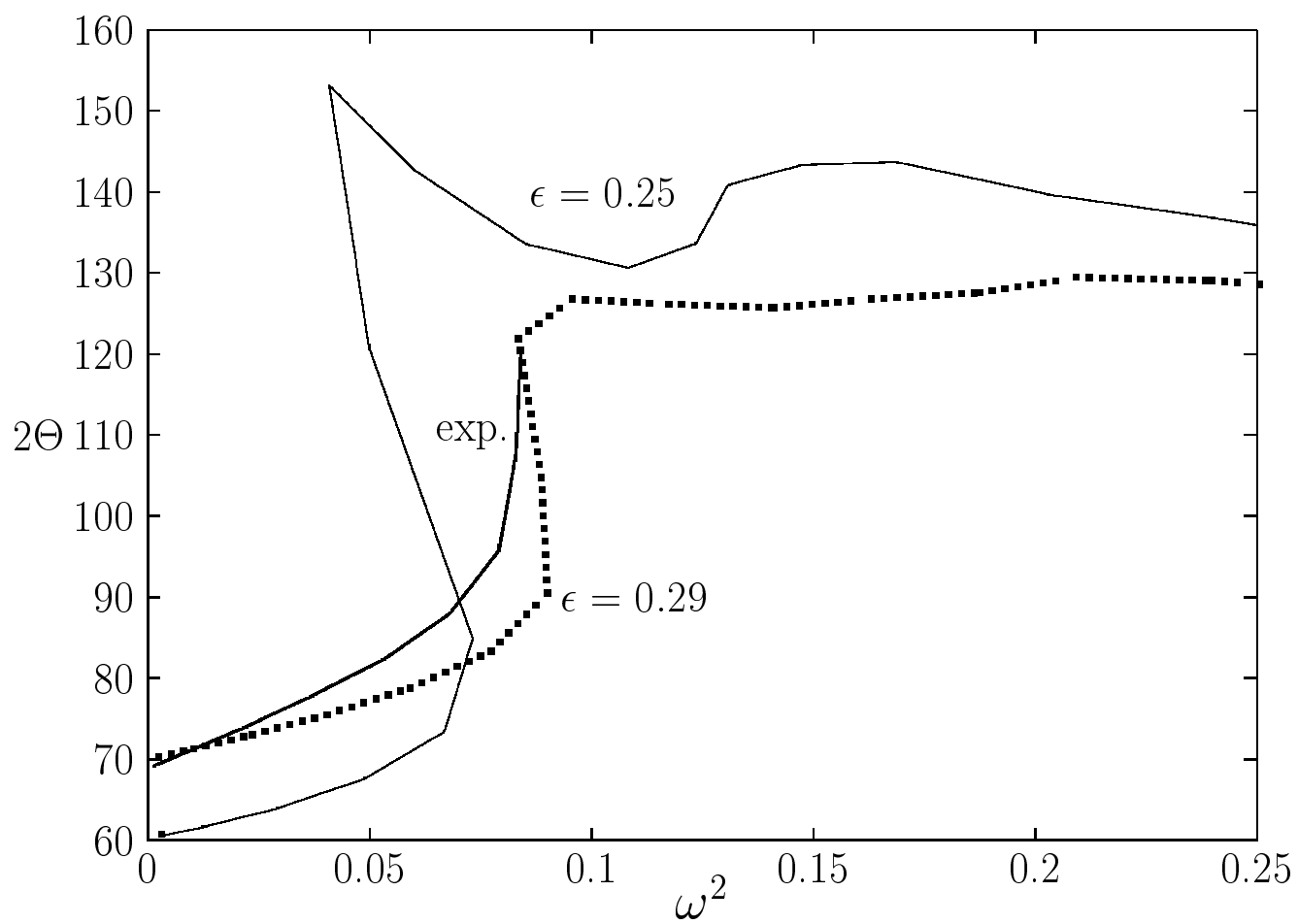


Figure 10: